XVI. *Observations on the Atmospheres of Venus and the Moon, their respective Densities, perpendicular Heights, and the Twilight occasioned by them.* *By* John Jerome Schroeter, *Esq. of* Lilienthal, *in the Dutchy of* Bremen. *Translated from the* German.

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On the Atmosphere of Venus.

ALTHOUGH the evidence afforded us by the most recent observations and discoveries, not only of the existence, but also of the various and singular properties of the atmospheres of Saturn, Jupiter, Mars, and the Moon, be in a manner incontrovertible; yet so inconclusive are the few observations hitherto made on the atmosphere of Venus, that several of the greatest astronomers* have lately thought themselves authorized to doubt its very existence.

The following inquiry may hence prove the more acceptable to those who favour the investigations of nature, not only as it evinces, in my opinion, the existence of such an atmosphere, but also as it points out several inferences concerning its nature and properties, which to me appear new and interesting.

What convinced me, twelve years ago, when I first began to observe Venus with a good three-feet achromatic telescope,

* See *Astronomie de* M. DE LA LANDE, §. 2272.

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that it actually has an atmosphere of no small extent, was the striking diminution of light which I noticed on the planet in its various phases from its exterior limb towards the interior edge of its illuminated surface, and especially near the latter: and this appearance it was which induced me to make farther observations on the subject, especially as I found that the phænomenon recurred as often as I looked at the planet with an HERSCHELLEAN four and seven-feet reflector, armed with the higher magnifying powers.

The great number of observations I have now made on this object, for a series of years, being upon the whole very similar in their nature and results, it would no doubt be not only tedious, but also superfluous, to describe them here at length: but the following general remarks it may be necessary to premise, in order to obviate all misapprehension, and the false conclusions that might be deduced from hasty and inaccurate observations.

In fig. 1. Tab. VI. which I have annexed for the sake of greater perspicuity, the light appears strongest at the outward limb *a*, *b*, *c*, from whence it decreases gradually, and in a regular progression towards the interior edge or terminator, and this not only towards its middle *d*, *e*, but also near the two cusps *f g*, *h i*, the light becoming so dim immediately at this border *g*, *f*, *d*, *e*, *b*, *i*, that it commonly loses itself in a faint bluish grey, forming a very indefinite ragged margin, scarce discernible with the best telescopes, and which, in several of the phases, resembles the interior uneven border of the moon, as it appears to the naked eye, or to a power magnifying from 1 to 4 times. The following circumstances must necessarily be attended to respecting these observations.

1. This diminution of light naturally shews itself between the greater eastern and western elongations and the inferior conjunction, when Venus appears about half full, or falcated, but still of sufficient breadth.

2. But this diminution is not equally sensible in all the phases of the planet, but sometimes much more so than at others. These appearances depend greatly on the clearness of our atmosphere, on the telescope giving a very distinct, soft, mellow image, and on the eye being properly prepared for such observations.

3. When, in the more falcated phases, we wish to observe distinctly and accurately this diminution of light at the farther extremities of the cusps, it is necessary to advert that, whilst about the middle of the terminating border between *d*, *e*, where the luminous part has its greatest breadth, this diminution, if seen under favourable circumstances, is too perceptible to leave the least doubt or suspicion, the points of the cusps, especially when the weather and other collateral circumstances are not favourable, appear, indeed, somewhat fainter, but yet almost as bright as the outward limb near *b*. As the diminution of light, whether the cause proceed from its own nature, or from that of the atmosphere, must be in the same proportion at the cusps as at the middle of the terminator, I could not but be puzzled at this appearance; but the multitude of observations I have since made on the points of the cusps, sometimes when they were tolerably bright, and at others when they appeared as faint as the other parts of the said border, at length cleared up my doubts, and led me to the true cause of this variable appearance, which, to prevent similar perplexities in others, I shall here state at length.

Considering the immense height of the mountains, and the great inequalities on the surface of Venus,* it is natural to suppose that, as at the times of its greatest elongations, one cusp frequently appears pointed and the other blunt, owing to the shadow of some mountain darkening the extremity of the latter, the same appearance may often take place in the falcated phase of the planet. But the cusp whose extremity is covered by a shadow, will, in this case, so far from appearing blunt, always exhibit a pointed appearance; it being in that aspect too sharp for us to distinguish accurately its true form. Should, for instance, the point of the cusp $\alpha\ \beta\ \gamma$, fig. 2, be immersed in a shadow from α to β, this point, in such a case, though it may be somewhat blunted from α to γ, will nevertheless appear sharp, because its whole breadth does often not exceed $\frac{1}{4}$ of a second, whence its true form must needs escape our observation. We do not, in this instance, see the very faintly illuminated point $\alpha\ \beta$, nor yet the real luminous edge $\gamma\ \beta$, but only its prominent and brighter part $\alpha\ \gamma$; a deception this, which cannot take place in the other parts of the terminator, where indeed the same causes, *viz.* the shadows of mountains, will, no doubt, at times occasion an uneven, ragged appearance, but cannot materially affect the very faint light of the whole.

It is, moreover, evident, that even when the extremities of the cusps are, as in fig. 1, not darkened by shadows, the faint luminous points $f\ g$, $h\ i$, are not always visible, but can only

* The perpendicular height of these mountains, as I have already shewn in a former work, is in proportion to the diameter of the planet, at least as great as the height of the mountains in the moon, in proportion to its much shorter diameter; this fact I have had frequent opportunities to ascertain. See Selenotop. Fragm. §. 522.

be seen with certainty at such times when the air is very clear and calm, and a good telescope with a large aperture, and high magnifying power, is used in the observation. And it follows that, except under those circumstances, the parts of the cusps *a f*, *c h*, must, on account of their great sharpness, be the apparent and sufficiently luminous extremities; since, according to well known principles, the exterior points *a*, and *c* will have as much light as the point *k*, near the middle of the crescent.

But in a clear and calm atmosphere, and with a high magnifying power, it is truly pleasing to see, after the eye is accustomed to it, how the whole of the terminating border, even to the farther extremities of the cusps, vanishes gradually, and becomes at last so faint, that in the day time, and where there are any inequalities, it insensibly loses itself in the colour of the sky. Such striking diminutions of light have I seen repeatedly with my four-feet reflector, with a power magnifying 280 times, and my seven-feet reflector, with a 370 magnifying power; and particularly on the 20th of November, 1791, when with a power of 161, I saw the light of the terminator dwindle away, and appearing, for a breadth of about 1 or $1\frac{1}{2}$ seconds, almost as grey as the ash-coloured spots on the moon.

Those who are at all acquainted with the theory of light, need hardly be reminded that on an illuminated spherical surface of a planet, the light will ever appear fainter towards its border, in proportion as the angle between the incident ray from the sun and the said surface becomes smaller.*

* It may equally be superfluous to illustrate how, since the sun appears on the planet Venus under a diameter of about 44′, its luminous border must have a no inconsiderable penumbra. It appears, however, from calculation, that this whole pe-

But what here claims our particular notice, is the singular circumstance that, except in the planet Mercury, concerning which I have not yet a sufficient number of observations, a similar diminution of light is not observed in so sensible a degree on *any of the other planets of* our solar system, our earth only excepted.

Mars, Jupiter, and Saturn, cannot, indeed, on account of their great distance, exhibit on our globe the variable phases of a half, or smaller portion, of an illuminated hemisphere, whence no fair arguments can be derived from those instances: but the comparative appearances of the moon, in this respect, will be thought the more singular if carefully attended to, the marginal diminution of light on this satellite, which, however, like Venus, is a sphere illuminated by the sun, not being nearly so perceptible and evident, as that above described. Of this we may fully persuade ourselves, by comparing the appearances of the terminating borders of the moon in its falcated phases or quadratures, with the same borders on Venus at the same periodical aspects. Should this striking difference not be reconcileable upon our established optical principles, nothing will remain but the analogy, that, as the density of our atmosphere checks the sun-beams the more, the longer they proceed therein in a direction which, after the rise or before the setting of the sun, carries them over a certain track of land; and as such a track, on which the sun at its rising and

numbra, which, according to the greatest apparent diameter of Venus, extends from 59 to 60″, measures in the direction perpendicular to the line of the cusps only 0,36″, or $\frac{1}{3}$ of a second of a degree; and that where this penumbra is observable in its exterior darker parts, its breadth is not above $\frac{1}{10}$ of a second, whence it cannot be visible throughout.

setting shines from the horizon, is but feebly illuminated; so also is Venus circumstanced with regard to the light it receives from the sun. To compare with some accuracy the intensity of light at the terminating borders on our earth, relatively to its full perpendicular illumination, with the same phænomenon on Venus, may, for want of opportunities to observe both planets at once from a proper distance, as may be done with regard to Venus and the moon, not be very feasible; but this, I think, I may with great confidence infer from my long series of observations, that Venus has an atmosphere in some respects similar to that of our earth, and which must far exceed that of the moon in its density, or power to weaken the rays of the sun.

Thus far had I proceeded in my observations, when the heavens favoured me with the following ones, which may prove the more interesting, as they not only confirm those hitherto made, but also lead to some farther inferences concerning the atmosphere of Venus.

Among all the favourable circumstances for observing the planet Venus, none could be more so than those I had in the months of March and April, but especially from the 9th to the 16th of March, 1790, when, besides the almost constant and unusual serenity of the sky, the planet, which was then in Aries, at 7° and 8° N. declination, was so high above the horizon, that, notwithstanding its approaching inferior conjunction on the 18th of March, at 4 P. M. I could still view it on the 16th, and should certainly have observed it during the conjunction, had not the weather become hazy on the 17th.

Under these very fortunate circumstances, I fell accidentally (not having, after ten years of constant attention to Venus,

been able to devise any new mode of observing) on the 9th, 10th, 11th, and 12th of March, upon an observation, which I repeatedly confirmed, and which, on account of its singularity, and the light it will probably throw on the physical constitution of this planet, will certainly be ever thought important; especially as it may not in many years be repeated under so favourable a combination of incidents.

On the 9th of March, 1790, immediately after sun-set, and till $6^h\,45'$, I saw Venus with a seven-feet reflector, magnifying 74,95, and 161 times, very distinctly, and uncommonly splendid. The southern cusp did not appear precisely of its usual circular form, but rather as is represented at *a*, fig. 3, inflected in the shape of a hook beyond the luminous semicircle into the dark hemisphere of the planet. This, however, after my former observations,* was not new to me: but a far more striking phænomenon, which I had never seen before, excited my admiration, and particular attention. The northern cusp was terminated at *b*, in the same narrow tapering manner as the southern, but did not extend in its bright luminous state into the dark hemisphere. From its point, however, the light of which, though gradually fading, was yet of sufficient brightness, a streak of glimmering bluish light proceeded into the dark hemisphere, from *b* to *c*, which though intermittent as to intensity, was yet permanent as to duration, and although very faint, could yet be plainly seen with both the abovementioned magnifying powers. Like the luminous line then seen on Saturn, its light seemed to twinkle in various detached points, and appeared throughout not only very faint, when compared with the light at the point of the cusp, but also of

* See Selen. Fragm. §. 523.

a very peculiar kind of faintness, verging towards a pale greyish hue.

The limb of the planet at this small part of its dark moiety, appeared with as faint a light, and, compared with the extremity of the southern cusp, as pale as the dark limb of the moon three days before and after the new moon, when it is faintly illuminated by the reflected rays from the earth: and it appeared to me, that towards the farther extremity *c*, where it was actually inflected, according to the circular limb of the dark hemisphere, its light vanished into a pale bluish tint, in the same manner as the more vivid light of the luminous hemisphere dwindles away towards the terminating border and the extremities of the cusps. The 3d fig. represents this striking phænomenon only in its projection, it being impossible to give a just representation of it in a drawing. Surprised at this new and singular appearance, I pointed it out to Mr. TISCHBEIN, an ingenious artist, who was then with me engraving the plates for my Selenotopographic Fragments, and desired him to attend particularly to all its circumstances.

He found and described the whole just as I had seen it. I made the drawing, fig. 3, under his eye; and we viewed the object together very attentively for about an hour, when the planet descended too low to be distinctly seen.

The apparent diameter of Venus was, by means of the projecting table, and the mean of several observations, all of which agreed to within 1″, found to measure 59″; but the greatest breadth of the illuminated part did not exceed 2,6″.

On the following evening, the air being as calm and serene as the preceding one, I observed the planet from 6^h to 6^h 40′, but on account of some necessary alterations in the apparatus

of the seven-feet reflector, I could use only the four-feet instrument, with powers magnifying 134 and 70 times. The southern cusp had its luminous prolongation, but not quite so distinct as the preceding night; but what was more remarkable, each cusp, but chiefly the northern one, had now most evidently a faint tapering prolongation, of a bluish grey cast, which, gradually fading, extended along the dark hemisphere, so that the luminous part of the limb was considerably more than a semicircle.

On the next night, being the 11th of March, when the seven-feet reflector was again fit for use, I found Venus before sun-set, with a power magnifying 95 times. At six o'clock, I saw distinctly the southern point terminating in a luminous streak; which now, as in the evening of the 9th, was longer and narrower than the bright termination of the northern cusp. The apparent diameter of Venus measured 60″; but the greatest breadth of the illuminated part could not be ascertained on account of some undulations in the air, occasioned by rising vapours: the light, however, sparkled through this tremulous medium, similar to the flash in a silver furnace; and comparing what I could distinguish with the table of projections, this breadth I found could not well exceed 2″. I also saw most distinctly the faint bluish luminous streak, sensibly extending in an inflected direction beyond the bright semicircle, and continued observing it till 6^h 30″, when, on account of the descent of the planet, I ceased my observation. The powers I used magnified 95 and 74 times.

No doubt can hence be entertained of the reality of this memorable phænomenon; but for reasons which I shall give hereafter, I was very solicitous to be able to measure the pro-

jection of this glimmering strip of pale light, and to ascertain whether in fact it only belongs to the northern cusp.

The very next, or the fourth evening, gave me a favourable opportunity for this purpose, and this was the more fortunate, as, on account of the hazy weather that immediately succeeded, of the increased twilight into which the planet advanced, and of its decreasing declination, this was the last evening in which such an observation could be made with some precision. At six o'clock, the atmosphere being uncommonly clear, I looked at Venus with the seven-feet reflector, magnifying 95 and 74 times. It appeared very distinct, and I ascertained, beyond the possibility of doubt, that the southern cusp projected somewhat into the dark hemisphere, and that from the point of the northern one, the very faint narrow streak of pale bluish light, intermittent in intensity on account of its faintness, but yet permanent as to duration, extended several degrees along the limb of the dark hemisphere of the planet. I strained all my visual powers, but could actually see this appearance only at the northern point. As the planet descended towards the horizon, the light of the very sharp southern, as well as that of the bright part of the northern point began to glimmer with gradual diminution, the latter more faintly than the former; but not the least appearance of a pale bluish prolongation could be traced at the southern cusp.

I had immediate recourse to the projection machine, and though I found it very difficult to measure, with the greatest accuracy, the very faint light at the northern, as well as the bright prolongation at the southern cusp, I succeeded, however, to ascertain

1. That the faint streak at the northern cusp, as represented in its projection, fig. 3. extended at least 8″ of a degree along the limb of the dark hemisphere.

2. That the prolongation of the southern cusp measured likewise full 8″, its inflected hooked form appearing now very distinctly: and,

3. That the apparent diameter of Venus subtended an angle of between 59 and 60″.

Had it not been for these very favourable observations from the 9th to the 12th of March, 1790, which evinced, beyond a possibility of doubt, that the illuminated limb of Venus actually exceeds the semicircle, I should, perhaps, never have arrived at the important cosmological discovery it led to; the days preceding them being too hazy, and those following, though somewhat clearer, yet too near the inferior conjunction of the planet to admit of fair observations. On the 17th, and on the 18th, when the conjunction took place at 4 P. M. the sky was cloudy. From the 21st to the 25th, as also on the 30th of March, the 1st, 7th, 8th, and 21st of April, the mornings about five o'clock were indeed sufficiently clear, and it may easily be imagined, that, considering the importance of the discovery, I did not fail to avail myself of them; but the northern declination of the planet had now decreased too much, and on the other hand, the twilight in our latitude was become too great to see distinctly and with certainty what I had observed before. I had no doubt, at times, of the prolongation of either cusp into the dark hemisphere; but circumstanced as the objects were, I was much upon my guard against visual deceptions. My journal, however, supplies me with the following appearances:

1. March 23d, 5^h 10′, $4\frac{1}{2}$ days after the conjunction, a faint light appeared, at times, playing along the dark limb from the southern cusp. The arc appeared hooked and irregular.

2. March the 24th, 5^h. I saw the exterior limb of the illuminated part manifestly much more convex at *a*, fig. 4. than at *b*. I also actually saw, at present, near *d*, a grey, faint, twinkling light, proceeding from the southern cusp in an inflected direction. Something of a similar appearance was likewise seen at the northern cusp *c*; but this was still fainter, and somewhat less inflected. The bright extremities of the cusps were very sharp, and similar in point of illumination.

3. March 25th, 5^h 15′. The bright hooked prolongations extending from both cusps, were not so perceptible as on the preceding days; but I was now the more struck on seeing, under the present circumstances, manifest traces of the same faint ash-coloured inflected streaks extending into the dark hemisphere, which I had beheld on the 10th of March, and which now proceeded from the extremities of both cusps, but more perceptibly so from the southern. See fig. 5.

4. March 30th, $11\frac{1}{2}$ days after the conjunction. I now saw no more of these appearances than I had done on the 9th and the preceding days of this month, particularly the 2d and 5th, when nothing of this phænomenon could be perceived.

As these observations, made after the conjunction, fully confirm those from the 9th to the 12th of March, I shall now proceed to state the inferences that appear to me evidently to result from them.

As there can now remain no doubt of the appearance of the pale ash-coloured streak of light, *b*, *c*, fig. 3. extending along the limb of the dark hemisphere of Venus; and as this planet

cannot be said, like the moon, to receive some light upon its dark side from our earth, or any other heavenly body, it follows that this light must either proceed immediately from the sun, which, as I have frequently observed concerning the high mountains Leibnitz and Doerfel in the moon,* throws its rays directly on the tops of very lofty ridges of mountains; or else that it is a light which partly illuminates the atmosphere of Venus, and partly, being reflected by this atmosphere, marks out by a faint glimmer the limb of the dark hemisphere of the planet, in the same manner as our morning and evening twilight acts upon ours.

All our present observations militate against the supposition of this phænomenon being the effect of light immediately proceeding from the sun; for,

1. This light does not appear, as on the mountains Leibnitz and Doerfel in the moon, in single, detached, and distant points; but as a continued streak of light, proceeding from the extremities of the cusps, and continuing along the limb of the dark hemisphere to a distance of about 8″, or, in proportion to the apparent diameter of the planet, no less than 15° 19′ of its circumference. This light also, compared with the bright part of the phase, is not unlike the comparatively pale limb of the dark part of the moon before and after its conjunction.

2. Were this the light of the illuminated summits of a chain of mountains, it would not appear so even, regularly connected, and spherical, as we behold it. But what removes all manner of doubt is,

3. The very different, extremely faint, bluish ash-coloured appearance of this glimmering light, which forms as great a

* See Selen. Frag. § 75. and Tab. IV. fig. 6 and 8.

contrast with the whitish more vivid light which is seen immediately on the cusps, as the ash-coloured light reflected from our earth on the dark limb of the moon does, when compared with the solar light on its phase. This pale light in the dark hemisphere, it must be owned, faded away towards its termination, in the same manner as the solar light did at the edge of the bright phase: but had this faint streak, like the more vivid light, been an immediate emanation from the sun, the gradual diminution would have been throughout progressive in a continued proportion; and the light in the dark part, immediately contiguous to the points of the cusps, must have had nearly the same degree of brightness as the points themselves, which was by no means the case.

Every circumstance, therefore, seems to evince that this phænomenon is occasioned by a light reflected by the atmosphere of Venus into the dark hemisphere of the planet, being in some measure the light of the atmosphere itself, when illuminated by the rays of the sun, or, in fact, a real twilight. But this will appear still more manifest when

4. We compare, according to the abovementioned observations, the alternate relative appearances of the cusps of Venus reciprocally with each other. On the 9th and 12th of March, 1790, when the southern cusp extended, not in the true spherical curve of the limb of the planet, but in a somewhat hooked direction, into the dark hemisphere, the pale bluish ash-coloured streak appeared only at the point of the northern cusp, from whence it proceeded, in a true spherical curve, along the dark limb of the planet. On the 10th of March, on the other hand, when the southern cusp did not penetrate so far into the dark hemisphere, the pale streak was perceived at both

points, though somewhat more sensibly at the northern than at the southern; and such also were the appearances after the inferior conjunction. These appearances will be thus explained by the effects of a twilight. The bright prolongation of the southern cusp, as it was seen on the 10th and 12th of March, *a*, fig. 3, must be ascribed to the solar light illuminating a high ridge of mountains situated at this region, whence it appears why this prolongation might not be strictly spherical. The twilight that must have existed at this part, would naturally be eclipsed by the much greater brightness of the light immediately derived from the sun, in the same manner as, on our earth, mountains that face the rising or setting sun, are known to darken the twilight that ought faintly to illuminate the regions situated immediately behind them. Were this not the case, there is no doubt but that a true spherical arch of the limb of the dark hemisphere would have appeared faintly illuminated: and such did we see was the effect of this twilight produced at both cusps when the bright prolongation was not considerable, the appearance being most decisive at that cusp where there was least of this prolongation.

Should this pale streak of bluish ash-coloured light actually be the effect of a twilight, it will not only prove the existence of an atmosphere, but we shall also, from the length of the streak, as measured on the 12th of March, 1790, be able to draw some inferences concerning its density, and real extent on the surface of the planet.

According to that observation, the apparent diameter of the planet measured 60″, and the visible stretch of the twilight, as far as it could be seen with a power of 95, applied to a

seven-feet reflector, was estimated at 8″, or eight of those parts of which 60 go to the diameter. This gives a circumference of 188,4, and the degree will be equal to 0,52 of these parts: and taking those 8″ as a chord, the arc, *b c*, fig. 3, over which the twilight extends, will measure 15° 19′; this being the portion of a great circle over which, in circumstances as favourable as those I was favoured with, the twilight will be seen extending over the dark hemisphere of the planet. This, and no more, is what I meant to express in one of my letters in which I mentioned the phænomenon in general, without any accurate computation, or particular induction.

But this pale luminous arc, *b c*, fig. 3, as it appears on the planet, not when it is at its greatest elongation, when we see half its illuminated surface, but at a time when it is approaching very near to its inferior conjunction, is in fact only the apparent, but by no means the real extent of the twilight, in a perpendicular direction east and west from the circle terminating the dark and illuminated hemispheres.

This real distance must evidently fall much short of the length of the abovementioned arc. The following is the method I have adopted to deduce the real extent of the twilight of Venus, from the apparent one as seen at the points of the cusps.

Let S, fig. 6, represent the sun, A D C B F the orbit of Venus, and T the place of the earth relatively to every aspect of Venus in its orbit. B will then be the point of inferior conjunction, C of the greatest western elongation, D of the superior conjunction, A of the greatest eastern elongation, and F nearly the point, not far distant from the inferior conjunction,

at which I observed the planet on the 12th of March, 1792, when I measured the extent of its twilight.

Let also A B G C fig. 7, be the hemisphere of the planet, turned towards our earth T, and A F D E C the other hemisphere turned towards the sun S, at the time of an inferior conjunction. And let A C represent the terminating surface perpendicular to the plane A F D E C G B extending from the centre of the sun through that of the planet Venus, and the eye of the observer on our earth. A *f c* C will then be the mutual edge of the two hemispheres, and the whole of the *dark one* A B G C *will* be directed towards the eye of the observer at T.

But whenever Venus, before or after an inferior conjunction, comes into such a position, relatively with the sun and our earth, as to be illuminated by the former in the direction *b* E *c*, fig. 7, which position, on the 20th of March, 1790, must have been nearly at F fig. 6; then will F *g c* G, fig. 7, be the mutual terminating surface of the dark and light hemispheres of the luminous circumference; of which surface, however, only one half, *c* G, can be seen from the earth; the other half, F *g c*, lying in the hemisphere that is turned away from the eye: *c* will be the point of the cusp seen from T.

This being premised, let the twilight of Venus be so considerable as to extend, in a gradually-fading progression, from the luminous terminating border F *g c* G, to the line *b f k i*, which let us assume as the real border of the twilight. Then will a part of it, K *f*, terminating at *f* in the direction *a g*, parallel to *b* E *c*, the central ray from the sun, be seen projecting into the dark hemisphere of the planet, and this prolongation, as it was seen on the 12th of March, 1790, will be the appa-

rent extent of the twilight; but its true extent will only be the distance $f g$ measured in degrees of a great circle. And it is sufficiently evident, that the smaller we assume the angle D c E, before or after an inferior conjunction, the more pointed of course will be its equal angle A c F, and hence also the longer the visible part of the twilight $f c$. It is likewise manifest, that when the elongation of the planet is equal to the angle D c C, the line D c k B will represent the terminating surface of the dark and light hemispheres, whence the planet will, at its greatest elongation, be illuminated by the sun in the direction C c, and the true extent of the twilight will coincide with the apparent one on the terminating plane A c.

In order then to calculate in any, except the greatest, elongation, the arc $f g$, in the right-angled spherical triangle $f c g$, or to ascertain the true extent of the twilight in degrees of a great circle, it will be requisite to measure from T, the apparent prolongation of the twilight into the dark hemisphere $f c$, and the apparent diameter of the planet. With these *data* it follows that, as the angle $f g c$ is a right angle, the angle $f c g$ = A c F = D c E may be found by calculation, and the arc $f g$ be thence easily deduced.

But the angle $f c g$ = D c E is equal to the complement of the angle S F T, fig. 6, on the planet, to 180°, or F S T + S T F, as is plainly illustrated by fig. 10, Tab. VII. Let S be the sun, T our earth, c the centre of Venus; draw $g c$ G perpendicular to S c K, and A c C perpendicular to T c: then is A c the projection of the visible terminating plane, as seen from T, and G g the projection of the luminous border. S c T is the angle on the planet, K c B its complement to 180°, and K c B = C c G;

For, $E\,c\,G = A\,c\,B = 90°$

$E\,c\,C = A\,c\,K$

And hence $C\,c\,G = K\,c\,B$.

To find, therefore, the angle $S\,c\,T$, calculate for the 12th of March, 1790, at six o'clock in the evening.

The true heliocentric long. of Venus	5^s	$18° 37' 35''$
Long. of the earth	5	22 21 20
Difference	3	43 45
Heliocentric lat. of Venus	3	23 0

Log. sin. $3° 43' 45''$	=	L. 9,9990795
+ Log. cos. 3 23 0	=	L. 9,9992424
Total = cos. $c\,S\,T$	=	9,9983219
	=	L. $5° 2' 0''$

Now in order to find the angle $S\,c\,T$, according to this heliocentric distance of Venus from the earth, we have in the triangle $S\,c\,T$, the angle at S, and the two sides $c\,S$ and $T\,S$, being the distances of Venus and our earth from the sun. We then find the log. $c\,S = 4857022$, and the log. $T\,S = 4997758$, and the angle at S being $= 5° 2'$; the sum of the two angles will be $= 174° 58'$, the half of their sum $= 87° 29'$, and then the computation will be as follows:

Log. T S = 4997758
Log. *c* S = 4857022

Log. tang. 10,140736
belongs to 54° 7′ 30″
Subtract 45

9° 7′ 30″
Log. tang. 9° 7′ 30″ = 9205803
Log. tang. 87° 29′ = 11357018

Log. tang. semid. 10,562821
Half the difference of the angles = 74° 41′ 15″
Half the sum = 87 29 0

Angle S *c* T = 162° 10′ 15″
Angle *c* T S = 12 47 45

Now the angle at Venus being = 162° 10′ 15″. Its complement will be = 17° 49′ 45″. But this is = the angle C *c* G = *f c g*, fig. 7, Tab. VI. and hence we farther derive the sine of the arc *f g* = sin. *f c* × sin. *f c g*; whence

Log. sin. 15° 19′ 0″ = 9,4218566
+ Log. sin. 17 49 45 = 9,4859768

Log. sin. *f g* = 8,9078334
which gives *f g* = 4° 38′ 30″.

Such, therefore, is the length of the arc of a great circle of Venus over which its twilight extends, as deduced from its appearance to the inhabitants of our globe under the most favourable circumstances: and if the true semi-diameter of the

planet be assumed = 834 geographical miles, then will the extent of the twilight, in the perpendicular direction from the terminating border, amount to 67 of those miles: and this extent of twilight on Venus, which appears to us to measure $4^\circ\ 38'\frac{1}{2}$, may, with great propriety, be compared with what LAMBERT calls the common twilight on our globe, when its border passes immediately through the zenith, at a time when he estimates the place of the sun at $6^\circ\ 23'\frac{1}{2}$ below our horizon.*

This computation affords also the following conclusion: according to the measurement of the 12th of March, 1790, the projection of the twilight, or fc, fig. 7, amounted in its whole length to 8″; whence sin. $fc \times$ sin. $17^\circ\ 49'\ 45''$, gives for the true extent of the twilight fc only 2,45 seconds of a degree; and hence it appears,

1. Why this twilight appears only at the points of the cusps and not at other parts of the terminating border, its extent, with its proportionably faint light, being too small; and this narrow faint light being, moreover, eclipsed by the superior brightness of the luminous hemisphere.

2. Why this twilight is distinctly seen only a few days before and after the inferior conjunction; the visible light of the luminous hemisphere being then only 2″ of a degree in breadth, and only the faint, declining, less luminous part of it apparent to the eye; whereas the projection of the twilight fc, fig. 7, will in this position be most extensive, and hence the more perceptible.

3. But should Venus at this time not be in the sign Aries, nor at so high a declination as when I saw it in March, 1790,

* LAMBERT's Photom. §. 998, *sqq*. Berl. Ephem. for 1776.

so that the sun do not rise and set much earlier and later than the planet; and should, at the same time, our own crepuscular light be very great; should the planet, I say, when the sun is some degrees below our horizon, have no great altitude, it will then be either not seen at all, or only appear in our strong twilight at or near the horizon. As, even besides all these favourable circumstances, it is moreover requisite that the atmosphere be uncommonly clear, it cannot be thought extraordinary that this very instructive phænomenon, on which I never reflect without particular satisfaction, should not have been seen by me sooner in the long and uninterrupted series of observations I have made, but it will rather be allowed to have been a most fortunate combination of incidents which enabled me to observe it for four days successively. It will also appear,

4. Why this phænomenon did not shew itself again after the conjunction with the same degree of splendour it did before; the northern declination of the planet being then daily decreasing, and the length and brightness of our twilight as gradually increasing. This will also point out the reason why it may not be seen again for some time. The fact, however, being now known to exist, there is no doubt but that some traces of it may, and I flatter myself will, be perceived, even under other perhaps less favourable circumstances.

This discovery, lastly, throws also some light on the constitution of the atmosphere of Venus, and the physical construction of that planet.

It appears, from the abovementioned observations, that however faint the last perceptible degree of evanescent light be in this phænomenon, we do not however, even with the

keenest eyesight, aided by the very best telescopes, distinguish the final termination of the twilight, but can only ascertain the border where its most luminous part begins to fade. If it be then mathematically demonstrated that this more luminous part, which may in some respects be compared to our own twilight, does actually, a few possible but trivial errors in the measurement excepted, extend 4° 38′ 30″ into the dark hemisphere, there can be no doubt but that this twilight does, in its whole extent, approach nearly to the extent of ours, which has been estimated at 18°; and that there is hence a material similarity between the two.

This may be inferred from the uncommon brightness of the solar light on this planet, and the comparatively very bright appearance of the more luminous measured part of its twilight, which also accounts for our seeing this more luminous part so distinctly even in the midst of our own twilight, and perhaps points out why even the pale bluish-grey light which in this, as well as in other proper phases, is seen to terminate the luminous part, forming an evanescent margin of 1 or $1\frac{1}{2}$ seconds, may afford some indication of a twilight gradually blended with the luminous terminating border. When this twilight was measured on the 12th of March, 1790, soon after six P. M. about twenty-six minutes after sun-set, the sun was at most only 4° below our horizon, and the angle *c* T S, fig. 10. Tab. VII. could, according to the above calculations, not measure above 12° 47′ 45″.

Hence it appears that, under such circumstances, the evanescent margin at the termination of the brighter visible part of the twilight of Venus, was at least as luminous as our own crepuscular atmosphere at that point of the heavens. And it

will also be easily perceived, with respect to the much stronger solar light on Venus, that if those last layers of its atmosphere, on which even the least vestige of the solar light can be traced, be transferred in idea into the atmosphere of our earth, admitting a similar density, but a less power of solar light, they would certainly exhibit no such perceptible traces.

And if it be probable, to a degree of evidence, that the twilight on Venus is nearly equal in its extent to that on our globe; if it be mathematically proved that its brighter part, which has so much light as to enable us not only to see it, but even to measure it across our fainter twilight, illuminates a zone 4° 38′ 30″ in breadth, we may hence surely infer the density and perpendicular height of at least the inferior more condensed part of the atmosphere, which is capable of reflecting such a crepuscular light; and apply those principles according to which the height of our own atmosphere, in as far, namely, as its lower strata are able to reflect the solar rays, and occasion a twilight, has been estimated at 34585 toises, or nearly nine geographical miles.

Let *a k b e f l*, fig. 9, Tab. VII. be the surface of Venus; *k b e*, an arc of it, which, according to the above calculation of the extent of the brightest twilight on the planet, is assumed at 4° 38′ 30″; *b d*, the height of that denser part of the atmosphere which actually occasions the twilight that is visible to us; and *b d*, the first ray of the sun, at this time 4° 38′ 30″ below the horizon, which is reflected by the highest stratum of the denser atmosphere *d*, upon the point *e* on the surface of the planet. We then have, in the triangle *d e c*, the side *c e*, or the semidiameter of Venus = 834 geogr. miles, the right angle *e*, and the angle *c* = 2° 19′ 15″, or half the angle

of the twilight. And the side $d\,c$ being to $e\,c$ as rad. is to the cosin. of the angle c, subtract from this $b\,c$, the semidiameter of Venus, and the remainder, $d\,b$, will be the height of the atmosphere. Now, taking 834 geogr. miles = 3178874 toises for the semidiameter, we have

Log. 3178874 = L. 6,5022632
Log. cos. 2° 19′ 15″ = L. 9,9996436

Log. $d\,c$ = L. 6,5026196 = 3181400
$-b\,c$ = 3178874

$d\,b$ = 2526

Hence it appears that the perpendicular height of the inferior and more dense part of the atmosphere of Venus, which has the power of reflecting the solar light to such a degree, as, under favourable circumstances, to be visible on our globe, where, with a good telescope, it assumes the appearance of a faint ash-coloured light, measures 2526 toises, or 15156 Paris feet.

As we are not acquainted with the laws of the inflection of light at the atmosphere of Venus, and are as much in the dark, concerning that as well as our own twilight, whether it be produced by a simple or compound reflection, this computation, it must be owned, is somewhat conjectural, and at best, conditional; and yet it supplies us with some farther means of estimating the height and density of the atmosphere of Venus with a somewhat greater precision.

If, according to the above principles and computations, it be determined that the twilight of Venus extends, at least in a considerable proportion, as far as ours; and if the inferior part

of the atmosphere, as far as a perpendicular height of 15000 feet, be so dense as to reflect so clear a light upon a zone extending 67 geogr. miles into the dark hemisphere, that we, at a distance of only 12° 47′ 45″ from the sun, and when this luminary is only from 3 to 4 degrees below the horizon, can see it so distinctly and evidently as to be able to compare it to our common twilight, measuring 6° 23′½, it must follow, that the atmosphere of Venus rises like ours, far above the highest mountains: and although we ascribe to that atmosphere the greatest possible transparency, it will still remain a more opake covering than, according to my Selenotopographical Observations, that of the moon appears to be. These observations also point out the probable cause why we do not see on this, our nearest, primary planet, even in the most favourable circumstances, any of the superficial shades, which, according to all analogy, we have reason to suppose on this, as well as on all the other planetary bodies; and which, considering the proximity of the planet, we should, when at its mean distance, and with a good HERSCHELLEAN telescope, magnifying 60 or 70 times, certainly distinguish nearly as plainly as I have seen, even with the naked eye, the superficial shades on the moon, not only the greatest ones, but also some of the lesser, such as Copernicus, Aristarchus, &c.; and surely, as well as we can discern, with a proper luminous magnifying power, the variable faint atmospheric stripes on Saturn and Jupiter, and the atmospheric opacities on Mars. At least it appears to me, that the density of the lower part of Venus's atmosphere, which enables it to reflect so clear a light into the dark hemisphere, sufficiently warrants such a conjecture: and it may well be conceived, that with so dense an atmosphere, the very

strong light of Venus must occasion a much greater degree of glare, than the soft light of the moon in a much more rarified atmosphere. In general it appears to me, that whoever will bestow some attention on these observations, and compare them impartially with the many others I have made on the atmospheres of the moon, Jupiter, Saturn, and even the sun, will find a striking coincidence between them.

The simple fact, that Venus in its different phases, and especially at the times of its greatest elongations, shews a much greater diminution of light at its terminating border than the moon, is, no doubt, sufficient to point out a much denser atmosphere than that of this satellite. And this is, moreover, fully evinced by the observations on its twilight. The circumstance also, that there are seen on this planet none of the flat spherical forms as are conspicuous on Jupiter and Saturn, none of the stripes or longitudinal spots parallel to the equator, which are seen on these planets and the sun, and which point out a certain stretch of atmosphere, gives room to infer, that the globe of Venus, with reference to its diameter, and other circumstances of physical arrangement in its construction,* performs its rotation round its axis in a much longer space of time than those planets, or the nearly similar ones of our earth and Mars; and this is actually confirmed by my observations on the diurnal period of Venus.

Thus we see a remarkable coincidence in every respect; and yet, though we cannot suppose a smaller, but rather a greater force of gravity on the surface of Venus than on our globe,

* These references have already been noticed by Chevalier DE GOIMPY, in the *Journal des Savans* for January, 1769; and before him, by M. DE MAIRAN, in the *Memoires Acad.* for 1729. See DE LA LANDE'S Astron. §. 3219, and 3122.

nature seems, however, to have raised on the former such great inequalities, and mountains of such enormous height, as to exceed 4,5, and even 6 times the perpendicular elevation of Cimboraco, the highest of our mountains. Thus are we, by these observations, led to a farther contemplation of the immense, and yet analogical variety with which the great Author of nature has dignified his works, as well in the greater objects, as in the smallest microscopic atoms; and the incessant novelty of combinations with which he has adorned them.

On the Atmosphere of the Moon.

Referring to my Selenotopographic Fragments for the proofs I there adduced of the real existence of a lunar atmosphere, which had been so frequently doubted; I shall also appeal to the same work for the observations I formerly made on several of its relative properties, compared with the same in our atmosphere, such as its greater dryness, rarity, and clearness, which, however, do not prevent its refracting the solar rays, having pointed out the circumstance, that the mountains in the dark hemisphere of the moon, near its luminous border, which are of sufficient height to receive the light of the sun, are the more feebly illuminated the more distant they are from that border: from which proofs of a refracting atmosphere, I also deduced the probability of the existence of a faint twilight, which, however, my long series of observations had not yet fully evinced.

As one fortunate discovery often leads to another, I had no sooner succeeded in my observations on the twilight of Venus, than I directed my attention to that of the moon, and applied

the calculations and inferences I there made, to some appearances I had already noticed on this satellite. It occurred to me, that if in fact there were a twilight on the moon, as there is on Venus and our earth, it could not, considering the greater rarity of its atmosphere, be so considerable: and that the vestiges of it, allowing for the brightness of the luminous part of the moon, the strong light that is thence thrown upon the field of the telescope, and in some measure the reflected light of our earth, could only be traced on the limb, particularly at the cusps; and even this only at the time when our own twilight is not strong, but the air very clear, and when the moon, in one of its least phases, is in a high altitude, either in the spring, following the sun two days after a new moon, or in the autumn, preceding the sun in the morning, with the same aspect: in a word, that the projection of this twilight will be the greater and more perceptible the more falcated the phase, and the higher the moon above the horizon, and out of our own twilight. This struck me the more, as I recollected having, two years ago, perceived such an appearance at the outward edge, near the points of the cusps, though I did not then reflect on the cause of it.

As all the requisite circumstances, however, do not often coincide, I thought myself particularly fortunate when, on the 24th of February, I was favoured with a lucky combination of them. Although this be as yet only a single observation of the sort, it is, however, in every respect so complete, and the inferences it leads to are, to me at least, so new and interesting, that I cannot withhold it from those liberal minded men, who are zealous in the pursuit of genuine philosophical knowledge.

On the abovementioned evening, at 5^h 40′, two days and twelve hours after the new moon, when in consequence of the libration, the western border of the grey surface of the Mare Crisium was 1′ 20″ distant from the western limb of the moon, the air being perfectly clear, I prepared my seven-feet reflector, magnifying seventy-four times, in order to observe the first clearing-up of the dark hemisphere, which was illuminated only by the light of our earth, and more especially to ascertain whether in fact this hemisphere, which, as is well known, is always somewhat more luminous at the limb than in the middle, would emerge out of our twilight at many parts at once, or first only at the two cusps. Both these points appeared now, most distinctly and decidedly, tapering in a very sharp, faint, scarce any where interrupted, prolongation; each of them exhibiting, with the greatest precision, its farthest extremity faintly illuminated by the solar rays, before any part of the dark hemisphere could be distinguished. But this dark hemisphere began soon after to clear up at once at its border, though immediately only at the cusps, where, but more particularly at their points, this border displayed, on both at the same time, a luminous margin, above a minute in breadth, of a very pale grey light, which, compared with that of the farthest extremities of the cusps themselves, was of a very different colour, and relatively as faint as the twilight I discovered on the dark hemisphere of Venus, and that of our own earth, when compared with the light immediately derived from the sun. This light, however, faded away so gradually towards the east, as to render the border on that side perfectly undefined, the termination losing itself imperceptibly in the colour of the sky.

I examined this light with all possible care, and found it of the same extent at both points, and fading away at both in the same gradual proportion. But I also, with the same caution, explored whether I could distinguish any part of the limb of the moon farther towards the east; since if this crepuscular light had been the effect of the light reflected from our globe, it would undoubtedly have appeared more sensibly at the parts most remote from the glare of the illuminated hemisphere. But, with the greatest exertion of my visual powers, I could not discover any part of the, as yet, wholly darkened hemisphere, except one single speck, being the summit of the mountainous ridge Leibnitz, which was then strongly illuminated by the solar light: and indeed eight minutes elapsed before the remainder of the limb became visible; when not only separate parts of it, but the whole displayed itself at once.

This alone gave me certain hopes of an ample recompence, and satisfied me that the principles I had laid down in my Selenotop. Fragm. § 525, *seqq.* concerning the atmospheres of the planets, and especially of the moon, are founded on truth. But a similar observation made on the 6th, after seven o'clock, afforded me several collateral circumstances, which strongly corroborate what I have there advanced on this subject. The whole limb of the dark hemisphere illuminated only by the reflected light of our globe, appeared now so clear and distinct, that I could very readily discern not only the large, but also the smaller spots, and among these Plato, Aristarchus, Menelaus, Manilius, Copernicus, &c. and even the small speck to the north-west of Aristarchus, marked *b*, Tab. XXVII. fig. 1. of the Fragments. I could apply the usual

power magnifying 161 times; and had full leisure, and the means to examine every thing carefully and repeatedly, and to take very accurate measurements.

Although a just idea of so delicate a phænomenon as this crepuscular light cannot possibly be conveyed by a drawing, but must be gathered from actual inspection, I have nevertheless attempted a delineation of it, and of the southern and eastern cusps fig. 1 and 2, Tab. VIII. as deduced from my measurements, especially at the southern cusp, in hopes thereby to render what I have farther to say concerning this observation the more intelligible.

The southern cusp, fig. 1. extended from *a* to *c*, with a gradually fading but still resplendent solar light, of its usual pale yellow colour, and terminated at *c* with a mountain. That this was really the point of the cusp, appears not only from the general construction of the falcated segment, which was sufficiently narrow even at its beginning *a*, near which it was somewhat disfigured at *b* by a high mountain; but also from the narrowness of its luminous curve at *d e* and *f*, the breadth of which seldom exceeded 1″, and had a sensible interruption so near as *d*. This curve was throughout, from *a* to *c*, except where the glare of the solar rays spread some degree of light, bordered with the pale ash-colour of the dark hemisphere, glimmering with the faint light reflected from our earth, out of which, however, rose the higher mountains *g*, *h*, *i*, *c*, which were now already illuminated by the sun; and farther on, not less than thirty lines, or, according to my usual projection, two minutes, distant from the point *c*, was seen another mountain *l*, which belonged to the high ridge Leibnitz, and also received its light immediately from the sun.

There can hence be no doubt of the termination of the cusp being at *c*: and this being well ascertained, I now distinguished with the greatest certainty the twilight extending from *c* to *k*. The most remarkable circumstances attending this light were, that it was broadest and brightest at *c*, and that it dwindled away and contracted towards *k*, where it lost itself in the faint glimmering of our terrestrial light; and that at the northern cusp, fig. 2, at which there do not appear to be so many mountains and inequalities as at the southern, this light exhibited the same pyramidal form, and was of equal length, and alike fading in intensity and colour, as that at the southern.

This light, compared with that of the thinnest and least bright part of the cusp *d c*, was as faint as the pale ash-coloured spots in the luminous hemisphere, when opposed to the bright ones. But this is still better illustrated by a comparison between the high mountain *l*, fig. 1, which now already appeared illuminated by the solar light, and the spot Aristarchus, which shone moderately merely with the light reflected from our globe. The said mountain had, comparatively with the thin luminous arc *d e f* of the bright hemisphere, and the mountains *g*, *h*, *i*, *c*, a very pale, fading, but yet brighter light than Aristarchus, as indeed might have been expected from what I said in my Selenotop. Fragm.; but this reflected light upon Aristarchus was however sensibly brighter than the glimmering light from *c* to *k*. And respecting the still fainter terrestrial light which bordered the luminous curve from *c* to *k*, fig. 1 and 2, I cannot give a better idea of it than by observing, that the light at the extremities of both the cusps appeared of a pyramidal form, similar to, but

though gradually fading, and very undefined, yet brighter than that of our zodiacal light, when, in the months of March and April, it blends itself, comparatively with the remaining colour of the sky, with the terrestrial light, terminating in a very sharp point.

The undefined and gradually fading appearance of this light was the cause that, though I had recourse to a dark projection table, I could not, however, take any accurate measurements of it. I found, nevertheless, by repeated comparisons, that the length of this pyramidal glimmering light, in which I could perceive no sensible inequality at the limb of the moon, amounted to about $\frac{2}{3}$ of the distance between the two mountains *c*, *l*, fig. 1, which shone with the solar light. Comparing also this southern twilight with the northern, it appeared of the same length; and on measuring the distance *c l*, I found it repeatedly $= 30$ lines $= 2'$; so that the length of the twilight must have amounted to 20 lines $= 1'\,20''$.

Its greatest breadth at *c* could, on the other hand, because of the extent and greater density of its light, be easily ascertained by means of the immediate application of the projection table. This measurement gave at most $\frac{2}{3}$ of a line, or full $2''$.

Although I be positively certain of this very remarkable appearance at both cusps, and of its perfect similarity, in all my observations, I could not, however, trace any vestige of a like crepuscular light at any other part of the terminating border: nor could I on the very next evening, being the 25th, and also on the 26th of February, perceive, even at the cusps, any of the twilight I expected to see there; the very thin, faint, luminous line which did indeed appear on the 26th, at the southern cusp between *a* and *b*, fig. 3, being undoubtedly the

effect of the immediate solar light, probably illuminating some prominent, flat area, as yet situated in the dark hemisphere.

Thus far the observations: and now for the application of them.

I need hardly insist upon the proofs, that the very faint pyramidal glimmering light, observed on the 24th of February, at the extremities of both cusps, could by no means be the immediate effect of the solar light, all the circumstances of the observations militating uniformly and decidedly against this supposition, which, were it true, would oblige us to admit a most unaccountable diminution of light, and thence also a density of the lunar atmosphere, that ought to exceed even the density of ours; a fact absolutely contradicted by all the lunar observations hitherto made. This light, indeed, was so very faint, that it disappeared at $7^h\ 20'$, when the moon approached the horizon; whilst, on the other hand, Aristarchus, which had no light but what it received from the earth, was still very distinguishable; and the summit of Leibnitz *l*, fig. 1, (which, though far within the dark hemisphere, was, however, illuminated by the immediate solar rays) displayed a degree of brightness which, although when compared with that of the cusp *d e f*, it appeared very faint and dwindling, equalled, however, that of our Pic of Teneriffe. Nor can it be conceived why this glimmering light broke off so suddenly at both the cusps, without a progressive diminution. It can hardly be supposed that similar, grey, prominent, flat areas, of the same form and dimensions, and comparatively of a faint light, which, whilst in the dark hemisphere, they derive immediately from the sun, exist on all parts of the moon; more especially as at the places observed, the limb happened to exhibit throughout

an exact spherical form, without the least sensible inequality; and as in both the bordering regions of the northern and southern hemispheres, especially in the latter, no such grey, prominent planes are any where discernible. It may then be asked, why did this faint glimmering light appear at both cusps, along equal arcs of the limb, of equal length and breadth, and of the same pyramidal form? and why did its farther extremity blend itself with the terrestrial light of the dark hemisphere, which, according to a great number of my selenotopographic observations, is by no means the case, even with those grey prominent areas which, being at some distance on the dark side of the terminating border, are nevertheless illuminated immediately by the sun?

These, therefore, could certainly not derive their light immediately from the sun; whence this appearance, like the similar ones on the planet Venus, can only be ascribed to the solar rays reflected by the atmosphere of the moon upon those planes, producing on them a very faint, gradually diminishing, glimmering light, which at last loses itself in the reflected terrestrial light, in the same manner as our twilight blends itself with the light of the moon. Every circumstance of the above observation seems to me to confirm this supposition; and hence the observation itself, which, though single, was however a most fortunate and complete one, must appear of no small degree of importance, since it not only confirms the observations and inferences on the long contested lunar atmosphere contained in my Selenotop. Fragm. but also furnishes us with many more lights concerning the atmosphere of planets in general, than had been afforded us by all those observations collectively.

This, and the mathematical certainty that the phænomenon is, in fact, nothing but a real twilight in the lunar atmosphere, will be farther evinced by the following theoretical deductions, in which the same mode of computing as was used in the observations on Venus, will be adopted, with this difference, that the angle *f c g*, fig. 7, Tab. VI. = A *c* F, is equal to the angle D *c* E, or the geocentric elongation of the moon from the sun.

1. According to the above observation, we find that the twilight at both cusps measured, from *c* to *k*, fig. 1 and 2, Tab. VIII. 20 lines = 1′ 20″. The apparent diam. of the moon being at 6^h P. M. = 31′ 18″; whence the twilight must have extended over an arc of the limb, measuring 4° 53′ 23″; and, if the real diam. of the moon be assumed at 465 geogr. miles, this extent must have answered to $19\frac{3}{4}$ of these miles.

2. But we have seen that *f c*, fig. 7, Tab. VI. or the measured projection of the twilight, is by no means the true distance of its farther edge *f*, from the terminating border *g*. This distance, therefore, must be found by the following computation:

Long. of the moon, Feb. 24, 6^h P. M.	=	12^s 7° 39′ 22″
Long. of the sun - - - - -	=	11 5 52 22
Difference of Long.	=	1 1 47 0
	=	31° 47′ 0″
Latitude of the moon	=	0 22 17
Log. cos. 31° 47′ 0″	=	L. 9,9294424
+ Log. cos. 0 22 17	=	L. 9,9999908
Log. dist. of ☽ from the ☉	=	L. 9,9294332
	=	31° 47′ 7″

Thus was the above the amount of the angle $f c g$, fig. 7; and as the sin. of the arc $f g$ = sin. $f c \times$ sin. $f c g$, then

Log. sin. $f c$ = L. 4° 53′ 23″ = L. 8,9306436
+ Log. sin. $f c g$ = L. 31 47 7 = L. 9,7215942

L. sin. $f g$ = 8,6522378
which gives $f g$ = 2° 34′ 25″

Whence I infer, that the breadth of the lunar twilight from the terminating border to where it loses itself in, and assumes the faint appearance of, the light reflected from our earth, measures, in a direction perpendicular to the aforesaid border, 2° 34′ 25″, equal to a breadth of 105 lines, or $10\frac{1}{3}$ geogr. miles.

3. Hence, admitting that the laws of the inflection of light are the same in the moon as on our earth and Venus, it will be easy to compute the perpendicular height of the inferior and more dense part of the lunar atmosphere $b d$, fig. 9, Tab. VII.; the angle c, in the right angled triangle $d c e$, = 1° 17′ 6″, and the radius $b c$ = 234 geogr. miles, or 891914 toises, being given; whence

Log. rad. » = L. 891914 = L. 5,9503210
− Log. cos. 1° 17′ 6″ = L. 9,9998908

5,9504302 = 892140
$- b c$ = 891914

$d b$ = 226

So that the inferior more dense part of the moon's atmosphere, measures, in perpendicular height, not more than 226 toises, or 1356 Paris feet; that inferior part, namely,

which reflects so much of the solar light into its dark hemisphere as produces a twilight, more luminous than the light reflected upon its surface from the earth, when it is only 31° 47′ 9″ from its opposition to the moon, when, consequently, it shews the greatest part of its disk illuminated, and affords some degree of light to the nights in the moon.

A cursory survey of these computations, shews that they not only contribute to illustrate the observations, but also that they coincide so fully with the collateral circumstances, as still farther to confirm those observations.

1. I was well aware, even at the time I made the observations, that the lunar twilight must, as it actually did, appear in the form of a lengthened pyramid. And indeed the 7th fig. Tab. VI. which represents the twilight *c k*, *g f*, F *h*, before the conjunction, projecting from the cusps towards the west, clearly points out, that only its pyramidal part *c f k* could be visible at T, whilst the whole remainder of it F *g c f h* extended over the whole hemisphere of the moon A F D E C, which was turned away from the eye of the observer.

2. It appeared to me, on the other hand, before I had sufficiently reflected on these collateral circumstances, that it could not well be reconciled to theory, that this twilight, whose length from *c* to *f* measured 20 lines, should at *c k*, where its breadth was the greatest, not be broader than $\frac{1}{2}$, or at most $\frac{2}{3}$ of a line. I soon, however, removed the difficulty by the following deduction. From the above computation, it appears that the side $f g = k c$, or the distance of *f k*, the farther edge of the twilight from the terminator *g c*, was = 105 lines, or an arc of 2° 34′ 12″. But the arc $k c = f g$, or the greatest visible

breadth of the twilight did not lay, like the side fc, along the edge of the limb, but nearly at right angles to it, in a direction immediately tending towards the eye: whence its orthographic projection could by no means amount to 10,5 lines, but must, according to well known principles, be less in the proportion of the sine of 2° 34′ 12″ to the cosine = 10,5, whence, by the following computation,

Log. $k\,c = fg$ = L. 10,5 = L. 2,0211893
× Log. sin. 2° 34′ 12″ - - = L. 8,6516648
———— = L. 0,5

This greatest apparent breadth of the twilight at the cusps could not, therefore, exceed 0,5 lines, or full two seconds; and this in fact was the breadth given it by the observation. It is scarce possible, that in the determination of so nice an object, theory should coincide more accurately with observation.

3. But it appears farther from these observations, why, notwithstanding all the pains I took, I could perceive nothing of this twilight at the terminating border of the falcated phase, and why, on the succeeding evening, this light disappeared likewise at the cusps. The brightness of the phase will, even on the first days after the new moon, occasion a strong glare in the field of the telescope, and this particularly at the spherical border, terminating the light; so that the margin contiguous to this border, which is only illuminated by the light reflected from the earth, may, indeed, appear luminous, but must necessarily be very undefined. The smaller the phase, moreover, the less will be the projection of the twilight at the terminating border, so that on the third day after the new moon it cannot well amount to much more than 2″ at the cusps; and it will then be so far obscured by the stronger

light of the illuminated hemisphere as to be almost evanescent. The best illustration of this, and of the known fact, that in the broader phases the twilight will, on account of the strong contrasting light of the luminous part, vanish even at the cusps, is given by the spots Manilius and Menelaus. Both are situated near the centre of the hemisphere turned towards our earth; both have considerable diameters, the one of 24 and the other of between 14 and 16 seconds, and both appear very distinctly on the third day after the new moon, with a degree of light rather surpassing that of the twilight. These spots, however, notwithstanding their great diameters, vanish on account of the increasing glare of light, so soon as four days after the new moon, although they be then still at some distance from the terminating border.* How could, therefore, a faint light, measuring only 2″ in breadth, be visible on the third day after a new moon, at the terminating border? And how could it at the fourth day still appear distinctly at the bright cusps, where it is, indeed, of a more considerable extent, but still must, in consequence of its pyramidal form, soon dwindle away into a very narrow tapering stripe? This, moreover, was precisely the case in the many observations I made on the planet Venus.

From the above calculations it also appears, that the lower and more dense part of the lunar atmosphere, that part, namely, which has the power of reflecting this bright crepuscular light, is only 1356 Paris feet in height; and hence it will easily be explained how, according to the different librations of the moon, ridges of mountains even of a moderate height, situated at or near the terminating border, may par-

* See Selenot. Fragm. §. 531.

tially interrupt, or, at times, wholly prevent this crepuscular light, either at one or the other cusp, and sometimes at both. I cannot hence but consider the discovery I here announce as a very fortunate one, both as it appears to me decisive, and as it may induce future observers to direct their attention to this phænomenon.

Admitting the validity of this new observation, which, I think, cannot well be called in question, I proceed now to deduce from it the following inferences.

1. It confirms, to a degree of evidence, all the selenotopographic observations I have been so successful as to make, on the various and alternate changes of particular parts of the lunar atmosphere. If the inferior and more dense part of this atmosphere be, in fact, of sufficient density to reflect a twilight over a zone of the dark hemisphere 2° 34′, or $10\frac{3}{4}$ geogr. miles in breadth, which shall in intensity exceed the light reflected upon its dark hemisphere by the almost wholly illuminated disk of our earth; and if, by an incidental computation, this dense part be found to measure 1356 feet in perpendicular height, it may, according to the strictest analogy, be asserted, that the upper, and gradually more rarified strata, must, at least, reach above the highest mountains in the moon. And this will appear the more evident, if we reflect that, notwithstanding the inferior degree of gravitation on the surface of the moon, which NEWTON has estimated at somewhat less than $\frac{1}{6}$ of that on our earth, the lower part of its atmosphere is, nevertheless, of so considerable a density. This considerable density will, therefore, fully account for the diminution of light observed at the cusps, and on the high ridges Leibnitz and Doerfel, when illuminated in the dark hemisphere; as

also for the several obscurations and returning serenity, the eruptions, and other changes I have frequently observed in the lunar atmosphere. This observation also implies,

2. That the atmosphere of the moon is, notwithstanding this considerable density, much rarer than that of our earth. And this, indeed, is sufficiently confirmed by all our other lunar observations. I think I may assert, with the greatest confidence, that the clearer part of our twilight, when the sun is 4° below our horizon, and when we can conveniently read and write by the light we receive from it, surpasses considerably in intensity the light which the almost wholly illuminated disk of our earth reflects upon the dark hemisphere of the moon $2\frac{1}{2}$ days before and after the new moon. But should we even admit an equal degree of intensity, it will, however, appear from computation that our inferior atmosphere, which reflects as strong a light over 4° as that of the moon does over 2° 34′ of their respective circumferences, must be at least eight times higher than that of the moon.

3. The striking diminution of light I noticed, in my twelve years observations on Venus, likewise indicates that the atmosphere of that planet, which is in many respects similar to ours, is much denser than that of the moon; and this will be still farther corroborated, if we compare together the several measurements and computations made concerning the twilights of different planets. There is no doubt but that the faintest twilight of Venus, as seen either before or after the rising and setting of the sun across our twilight, is much brighter than that of the moon; and it appears, moreover, from computation, that the denser part of the atmosphere of Venus measures at least 15000 Paris feet in height, and spreads

its twilight 67 geogr. miles into the dark hemisphere, whilst the denser part of the lunar atmosphere, whose height does not exceed 1356 feet, produces a faint twilight not above $10\frac{1}{3}$ geogr. miles in breadth. Thus, as my successful observations on the twilight of Venus led me to the discovery of that of the moon, so did these latter reciprocally confirm the former: and thus, which ever way we contemplate the subject, must we be struck with the coincidence that prevails throughout.

4. But if the lunar atmosphere be comparatively so rare, it follows that the inflection of light produced by it cannot be very considerable; and hence does the computation of M. DU SEJOUR, according to which, the inflection of the solar rays which touch the moon, amounts to no more than $4''\frac{1}{2}$, receive an additional degree of authenticity.* Besides which,

5. As the true extent of the brightest lunar twilight amounts to $2^\circ\ 34'$, the obliquity of the ecliptic in the moon only to $1^\circ\ 29'$; the inclination of the orbit of the moon, on the contrary, to $5^\circ\ 15'$, and its synodic period, during which it performs a revolution round its axis is $= 29^d\ 12^h$; it follows, that its brightest twilight, to where it loses itself in the light reflected by the almost fully illuminated disk of our earth, must, at least at its nodes, last $5^h\ 3'$, and that it will be still longer at other parts of the orbit, according to the situation of the nodes.

6. And lastly, it being a well known fact* that the fixed stars, as they approach the moon, diminish in splendour at the most only a very few seconds before their occultations, it was

* See DE LA LANDE's Astron. §. 1992—1994.

† See Selenot. Fragm. §. 531, with its note.

natural for me, after the successful observations I had made on the twilight of the moon, to pay particular attention to this circumstance. On the 25th February, at 6^h P. M. the sky being very clear, the limb of the dark part of the moon appeared uncommonly distinct; and only a few seconds of a degree from its edge was seen a telescopic star, of about the 10th or 12th magnitude. I counted full 20″ before its occultation, and $18\frac{1}{2}$ of these, without the least perceptible diminution of light. The star, however, began now gradually to fade, and after the remaining $1''\frac{1}{2}$, during which I observed it with all possible attention, it vanished in an instant. This observation agrees perfectly with the above computations. Although it be proved that the inferior dense part of the lunar atmosphere reflects a stronger light than that which the dark hemisphere receives from an almost fully illuminated disk of our earth; and although, considering the inferiority of gravitation on the surface of the moon, there be no doubt that this dense part, together with the superior gradually more rarified regions of its atmosphere, must extend far above its highest mountains; it is yet a fact that the breadth of this observed twilight, to where it loses itself in our reflected terrestrial light, does not measure more than 2° 34′: it is, therefore, highly probable that its greatest extent, in the most favourable phases near our new moon, can never exceed the double of the above arc, or 5° 8′; and hence we can only infer a perpendicular height of an atmosphere, capable of inflecting the solar rays, which at most measures 5376 feet: nor is it very likely that, unless accidental, and hitherto unknown circumstances should occasionally condense different parts of this atmosphere, these upper strata should materially affect the distinctness of a star seen through it.

But admitting the height of the atmosphere, which may affect the brightness of a fixed star, not to be less than 5376 feet, this will amount to an arc of only 0,94″, or not quite one second; and as the moon describes an arc of 1″ in 2″ of time, it follows that in general the fading of a star, which approaches to an occultation, cannot last quite 2″ in time; that if the appulse be at a part of a limb of the moon where a ridge of mountains interferes, the gradual obscuration will last a still shorter time; and that it may, under some circumstances of this nature, be even instantaneous.

Supplement to the foregoing Observations on the Atmosphere of the Moon.

The following observation of an occultation of Jupiter by the moon, when near its full, occurred to me by mere accident, on the 7th of April, 1792; and I was the more gratified by it, as I had long wished for an opportunity to observe such a phænomenon, and had no expectation of seeing it at this time, no mention being made of it in the Ephemerides.

The sky being very serene, and Jupiter uncommonly bright, I prepared my seven-feet reflector, magnifying 74 times, in hopes that the strong light and distinctness it afforded would enable me to compare the appearances of this phænomenon with the results which I had deduced from my late observations on the height and density of the atmosphere of the moon.

Fig 1. Tab. IX. represents the situation of Jupiter's four satellites, as they appeared, most distinctly, two of them to the westward, the second about one, and the first near two of Jupiter's diameters distant from its limb; and the two others

to the eastward, the third about seven, and the fourth near eight of the same diameters, distant from the said limb.

Fig. 2. shews Jupiter, with its belts, and of a somewhat spheroidical form, as it now appeared to me, and as distinctly as I had ever seen it. The equatorial belt, from *a* to *d*, was very apparent. It consisted properly of two zones, *a b*, and *c d*, of a brownish-grey colour, with a more luminous interval *b c*, between them. At *e* and *f* were two comparatively well defined stripes, which I had noticed for many years back, but which now crossed the whole disk; and the polar regions appeared again, from *g* and *h*, more dim and grey than the bright part of the planet. But what particularly struck me, were two nebulous undefined spots, *i* and *k*, which were sensibly darker than the principal zone *d d*; and at *l*, a still more remarkable, circular, though imperfectly defined spot, somewhat brighter than the luminous interval between the zones, and perfectly similar to the remarkable luminous spot which I had observed in 1786 and 1787, on the same part of Jupiter, and which then led me to some very unexpected inferences concerning the atmosphere of that planet.*

These favourable circumstances led me to the following accurate observation, which I was certain would prove instructive to me. At $10^h\ 40'\ 50''$, I saw the spot *i* at about the middle of its parallel; and immediately after began the occultation, than which a more distinct and beautiful one was, perhaps, never seen.

* *Beytr. zu den neuesten Astron. Entd.* p. 75. and Tab. I. fig. 6.

Immersions.

The western, preceding, first satellite disappeared behind the sharp bright limb of the moon, at $10^h\ 43'\ 12$.*

N. B. There may here have been an error of two or three seconds, as the satellite became indistinct at the limb.

The second satellite disappeared, without becoming at all indistinct, exactly at $10^h\ 45'\ 19''$.

The western limb of Jupiter came in contact, most distinctly, with the eastern limb of the moon, at $10^h\ 46'\ 32'',5$.

Jupiter's eastern limb disappeared, as distinctly, at $10^h\ 48'\ 20'',5$.

N. B. This immersion took place, as represented in fig. 3, to the eastward of Aristarchus, at about the 24th degree of N. lat.

The third satellite disappeared, after having been for about one or two seconds faint and indistinct, at $10^h\ 58'\ 57'',5$.

The fourth satellite, which appeared the least of them all, became undiscernible near the limb, and vanished at about $11^h\ 2'\ 16''$.

N. B. There may have been an error of four or five seconds.

Emersions.

The two preceding first and second satellites were here likewise of use in determining precisely the emersion of both the limbs of Jupiter, from the dark hemisphere of the moon.

* True time, determined by equal altitudes, taken on the 12th and 8th of April, and 2d and 27th of March, and by the known rate of the clocks; but as no altitudes could be taken on the 7th, there may be an error of two or three seconds.

The first appearance of Jupiter's western limb was very distinct at $11^h\ 43'\ 54''$.

Emersion of the eastern limb, as distinct, at $11^h\ 45'\ 39'',5$.

N. B. This emersion took place, as represented in fig. 4, to the north-eastward of Seneca (B, Tab. VIII. of the Frag.), at about the 23d degree of N. lat.

The emersion of the next, or third satellite, was not observed.

That of the fourth was distinct, at $11^h\ 59'\ 1''$.

This observation gave me the more satisfaction, as it singularly contributed to confirm the discovery I had been so fortunate as to make of the twilight in the moon, and the height and density of the lower stratum of its atmosphere.

Experience has sufficiently proved, that a stronger will ever obscure a fainter light; and it follows hence, that the light of a bright star approaching the moon, when full, or nearly so, will lose something of its lustre: but little can be inferred in favour of an atmosphere either of the moon, or of Mars, from the observation of CASSINI, in which, as Dr. HERSCHEL has illustrated by some observations of his own,* a star in Aquarius, of the sixth magnitude, and as yet six minutes distant from Mars, diminished in light when both were seen in the same field of the telescope. A mere apparent diminution of light, occasioned by the glare of a larger luminous object, when seen at the same time with a smaller one, in the field of the telescope, is one thing; and another thing is a real indistinctness of the small luminous body, which increases in proportion as they approach nearer to each other.

* Phil. Trans. for the Year 1784, Part II. p. 271.

It was very natural for Jupiter to diminish in brightness, when it approached so near to the moon, then almost at its full, as to be seen at the same time in the field of the telescope, which was, in fact, the circumstance of this observation; but I could not observe any progressive variation of light in the eastern and western, equally luminous, disks, proportional to their distances from the limb of the moon, much less a real indistinctness; and this neither when the limbs of the two planets were nearly in contact, nor when Jupiter was partly, or about one half, covered by the moon

It was a sight truly gratifying to an eye accustomed to the light of the moon, or in general to similar observations, to behold how Jupiter, at its immersion, as well as emersion, being half, or more than half, covered by the moon, exhibited its belts and other parts, as distinctly, close to the limb of the moon, as it does at some distance from it: and had I not already succeeded in my numerous observations on the atmosphere of the moon, and very recently in those which enabled me to determine its twilight, I should perhaps have adopted the doubts the ancient astronomers entertained concerning the existence of a lunar atmosphere; and this the rather, as when Jupiter, in its immersion, was so far covered, that the luminous spot *l*, fig. 2, was close to the moon, I could plainly distinguish this spot, although it be in itself by no means very perceptible.

Such, however, must have been the appearances, according to my new observations and measurements of the twilight of the moon: for if it be proved that the extent of this twilight, to where it loses itself in the light reflected from the almost wholly illuminated disk of our earth, amounts to no more

than an arc of 2° 34′ of the circumference of the moon, and if it be hence demonstrable that its greatest dilatation does barely amount to 5° 8′, and the perpendicular height of that part of the lower more condensed stratum of its atmosphere, which is capable of reflecting the solar rays, and of producing some other, perhaps more remarkable, obscurities in the stars seen through it, *does not exceed 5000 Paris feet*, and hence cannot reach above one second of a degree above the limb of the moon; we need not wonder that so small a magnitude, which loses itself in the inequalities of the limb, many parts of which are known to be considerably mountainous, should not become sensible, especially at the approach of a body of so large a diameter as Jupiter, and when so small a magnifying power is applied. And thus may I, with confidence, assert a perfect coincidence between this and my many other observations.

The appearance, fig. 5, when Jupiter, at the emersion, the objects being particularly sharp and distinct, came forth from behind the moon, which now covered no more than one-quarter of its diameter, was truly splendid and satisfactory; and I must here particularly mention the circumstance, that the part of the moon's dark hemisphere, between its bright terminating edge *m n*, and its outward limb, bordering upon the emerging planet, *o p*, was particularly opake, and hence produced a very striking effect.

I omit entering here upon any farther considerations, and shall conclude with observing that, after the occultation was completely ended, the luminous spot *l* had at 12^h 1′ so far advanced in its parallel *d e*, as to have reached to within $\frac{1}{6}$, or at most $\frac{1}{5}$, of its whole length, of the western limb. And that

Fig 1.

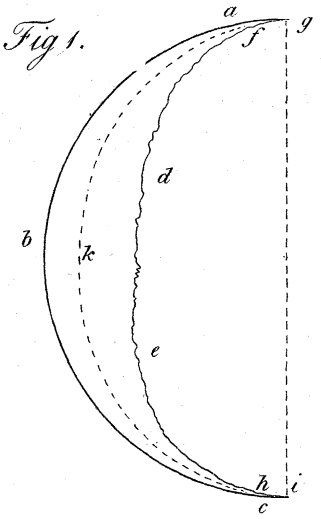


Fig 2.

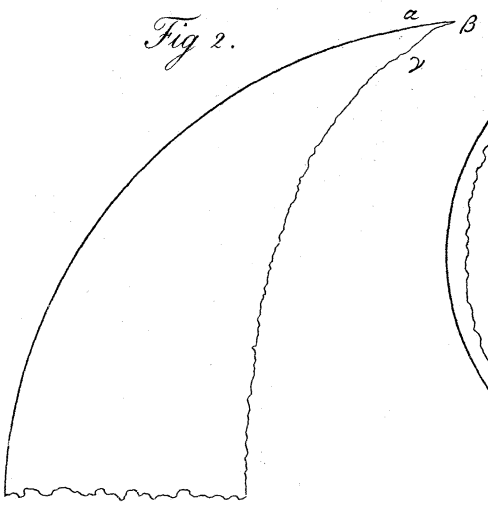


Fig 3.

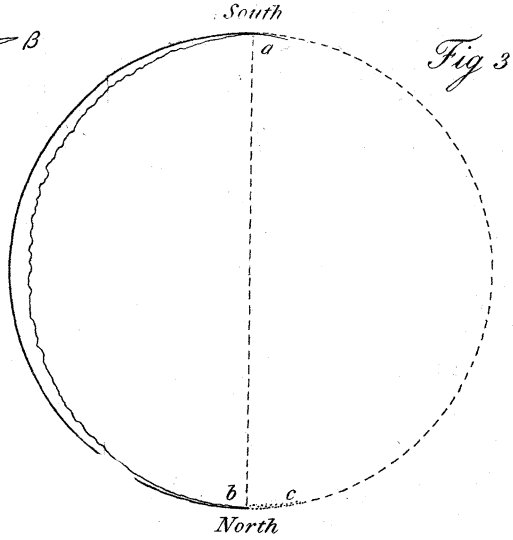


Fig 4.

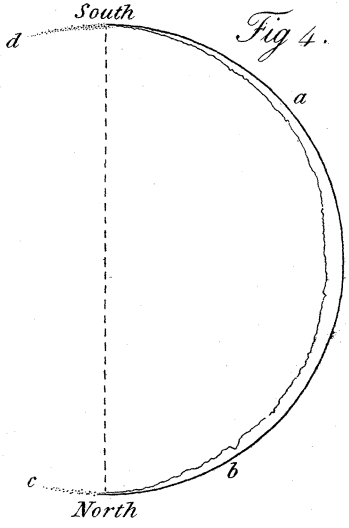


Fig 5.

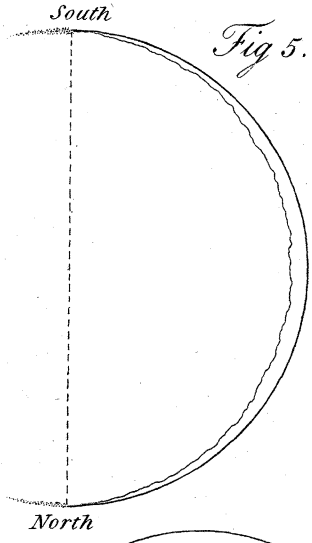
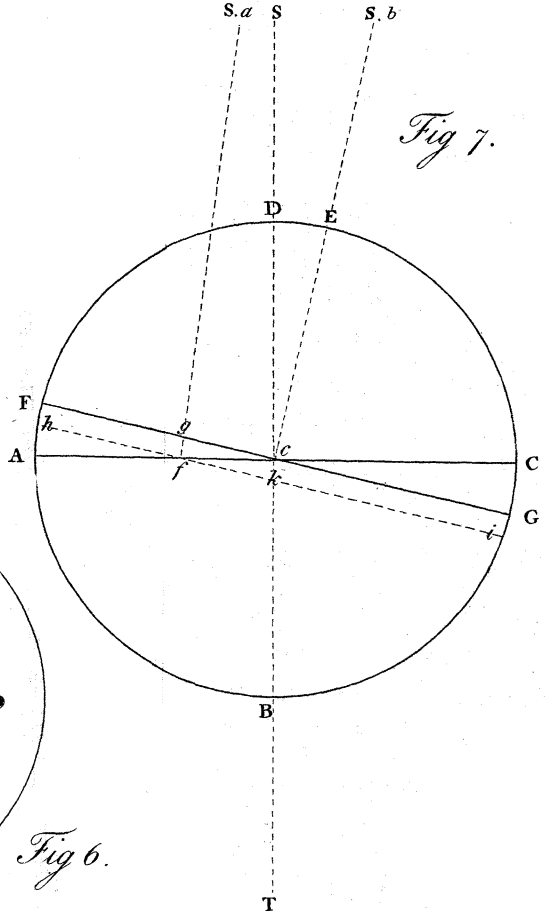


Fig 7.



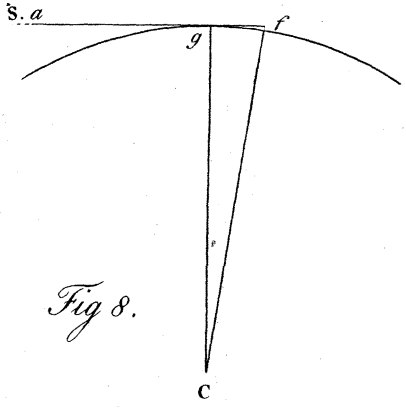


Fig 8.

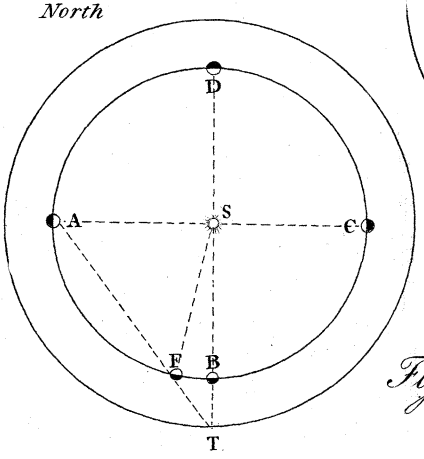


Fig 6.

Fig 9.

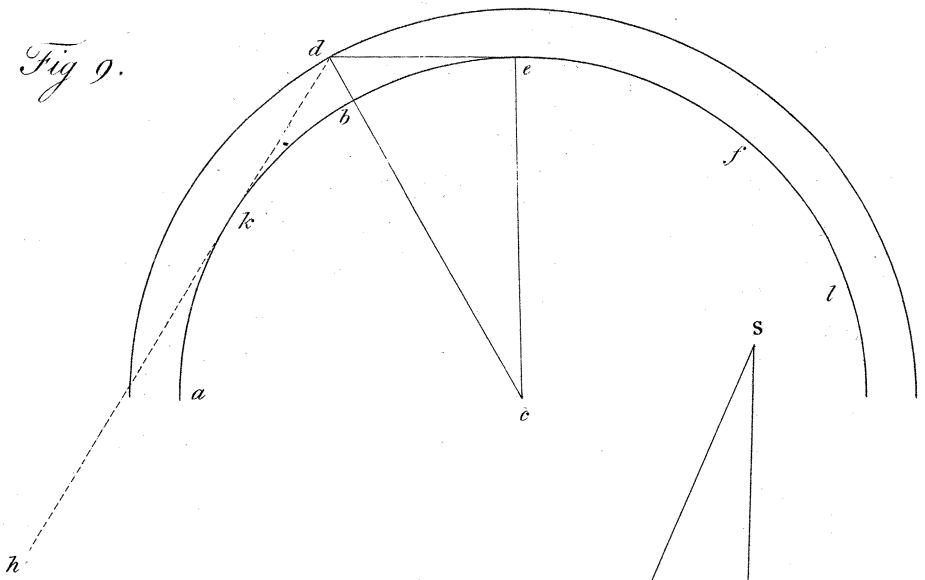
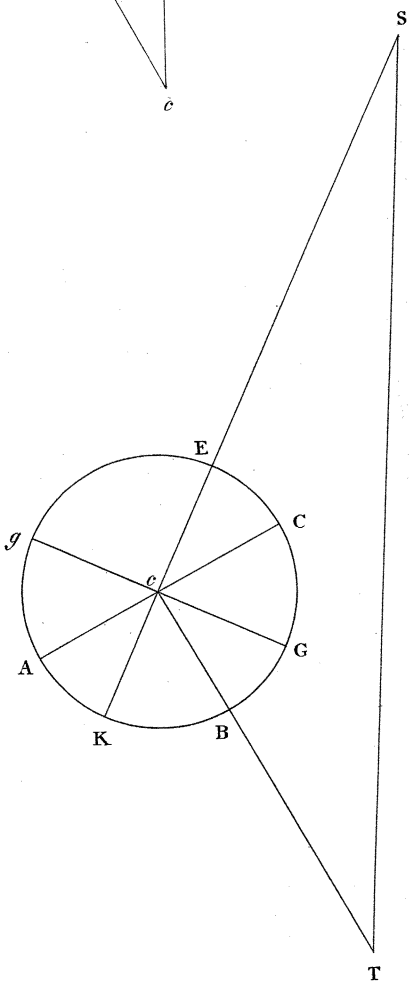
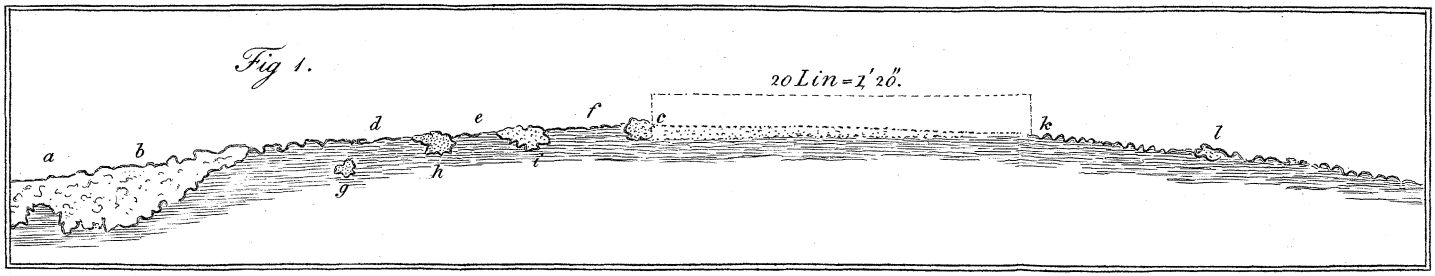
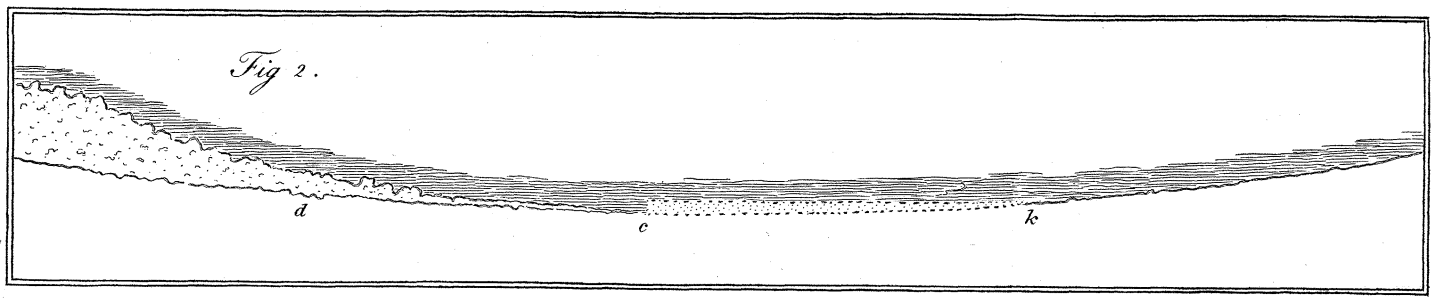


Fig 10.







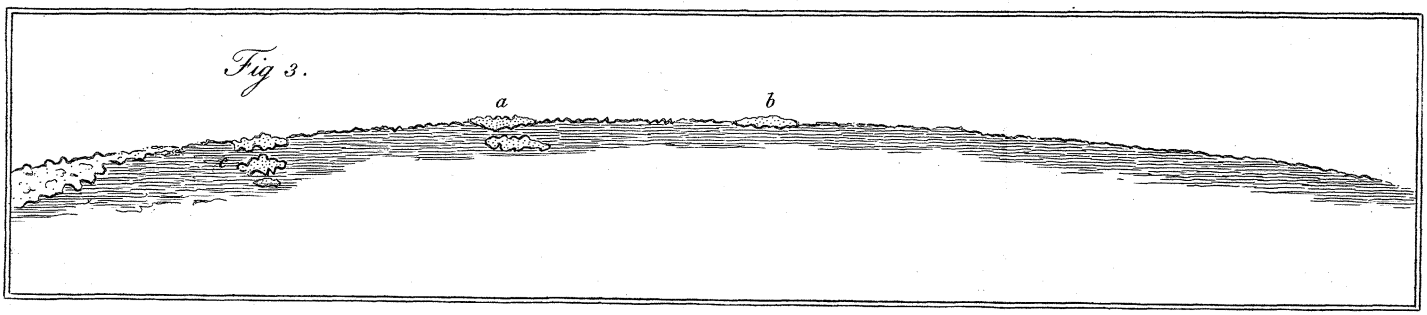


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Fig 1.

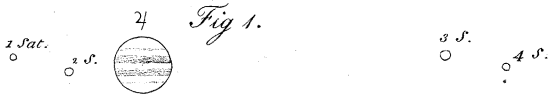


Fig 3.

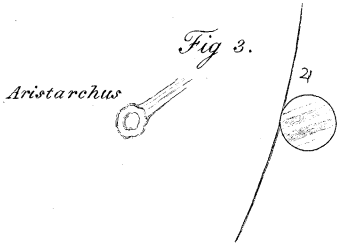


Fig 4.

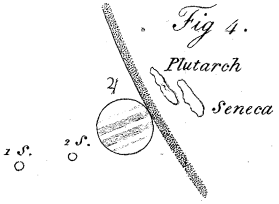


Fig 2.

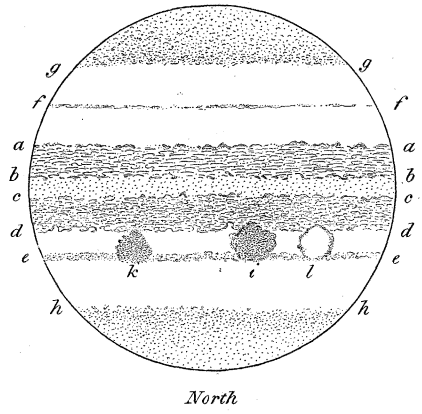
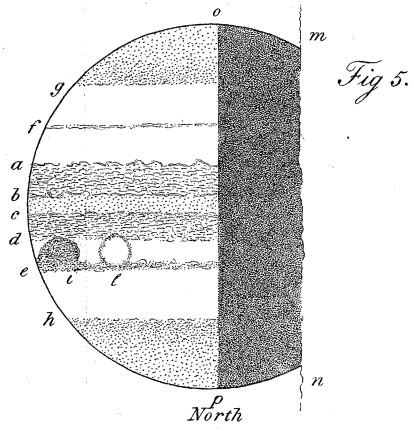


Fig 5.



on the 28th of March, five days after a new moon, I observed an occultation of a very distinct, though telescopic star, by the dark hemisphere of the moon, in which, agreeably to the above observation, not the least gradual diminution of light, or indistinctness could be perceived, the star being seen to vanish on a sudden.

Lilienthal,
April 10, 1792.